

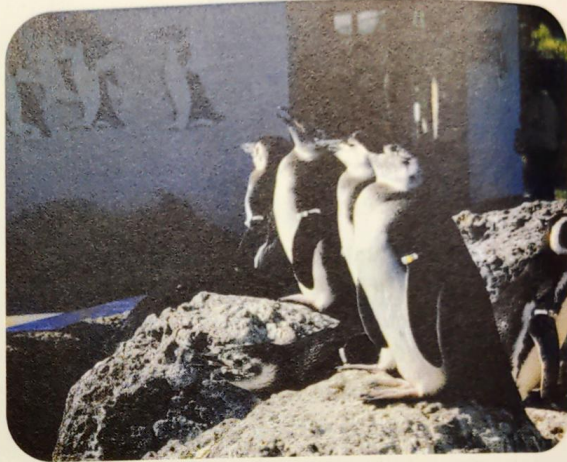
6.1

Exploring Integer Addition and Subtraction

▶ GOAL

Add and subtract integers.

Explore the Math



After being repaired, a zoo's Tropical Forest Exhibit and Antarctica Exhibit were at room temperature, 20°C .

To prepare for exhibiting plants and animals again, the temperature in the Tropical Forest Exhibit was warmed at a constant rate to 28°C . The temperature in the Antarctica Exhibit was cooled at a constant rate to -1°C .

The temperatures were checked after 4 h, 5 h, and 6 h, and recorded as follows:

Time (h)	Temperature in Tropical Forest Exhibit ($^{\circ}\text{C}$)	Temperature in Antarctica Exhibit ($^{\circ}\text{C}$)
0		
1		
2		
3		
4	24	8
5	25	5
6	26	2
7		
8		

Communication Tip

- The positive sign (+) is not usually included with positive integers.
- Brackets are only needed when the sign of a number follows an operation symbol and might be confused with the operation symbol. For example, $(+3) + (-6) - (+4)$ can be written as $3 + (-6) - 4$.

? How can you use integer operations to determine the missing temperatures in the table?

- A. Copy and complete the table.
- B. Look at the temperature pattern in the Tropical Forest Exhibit. How did the temperature change each hour? Express your answer as a rate, using $^{\circ}\text{C}/\text{h}$.
- C. Look at the temperature pattern in the Antarctica Exhibit. How did the temperature change each hour? Express your answer as a rate, using $^{\circ}\text{C}/\text{h}$.
- D. Explain why the equation $24 + 3 = \blacksquare$ can be used to calculate the temperature in the Tropical Forest Exhibit after 7 h.
- E. Write an equation that can be used to calculate the temperature in the Antarctica Exhibit after 7 h.
- F. Write an integer addition or subtraction equation that can be used to calculate the temperature in each exhibit
 - a) 3 h after the temperature started changing
 - b) 2 h after the temperature started changing
- G. Suppose that the temperatures in both exhibits continued to change at the same rates. What would the temperature be in each exhibit 12 h after it started changing?
- H. Suppose that the temperature in the Antarctica Exhibit continued to change at the same rate. If you knew that the temperature in the Antarctica Exhibit was -1°C , what equation could you use to determine the temperature 5 h later?

Reflecting

- 1. Why did you need to know the rate at which the temperature changed in each exhibit before you could calculate the temperatures that were not recorded?
- 2. How could you use a pattern, a number line, or integer calculations to help you complete the table?
- 3. How could you use integer calculations to calculate the temperatures in steps E and F?

6.2

Relating Integer Subtraction to Addition

- You will need**
- number lines
 - coloured pencils
 - red and blue counters
 - a calculator

▶ GOAL

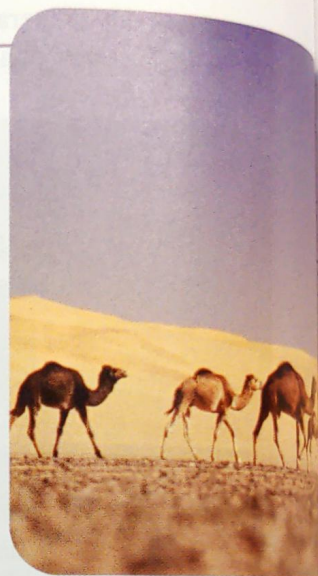
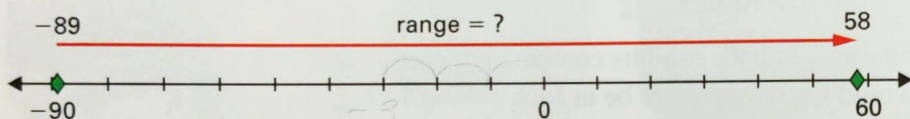
Subtract integers by measuring the distance between them.

Learn about the Math

The highest temperature ever recorded was 58°C in Libya in 1922.
The lowest temperature ever recorded was -89°C in Antarctica in 1983.

? What is the range of these temperatures?

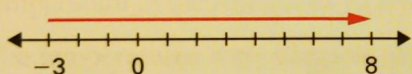
Ken wrote $58 - (-89)$ but was not sure how to complete the calculation. Nathan suggested using a number line. Ken said that the numbers were too great to count the spaces.



Example 1: Using a number line to solve a simpler problem first

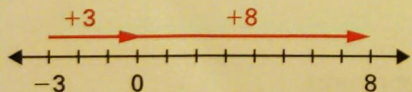
What is the difference between 58°C and -89°C ?

Nathan's Solution



$$8 - (-3) = 8 + 3$$

$$= 11$$



$$58 - (-89) = 58 + 89$$

$$= 147$$

The range is 147°C .

First I subtracted $8 - (-3)$ using a number line.

To subtract -3 from 8 , I drew an arrow to show what to add to -3 to get to 8 . Since the arrow goes to the right, it represents a positive number. So I coloured it red. The arrow is $3 + 8 = 11$ units long.

The part of the arrow from -3 to 0 is 3 . The rest of the arrow, from 0 to 8 , is 8 .

Subtracting $8 - (-3)$ has the same result as adding $8 + 3$.

It makes sense that subtracting an integer has the same result as adding its opposite. On a number line, the part of the arrow from -89 to 0 is 89 . The rest of the arrow is 58 .

$58 - (-89)$ must equal $58 + 89$.



Example 2: Using counters to solve a simpler problem first

What is the difference between 58°C and -89°C ?

Ken's Solution



$$(8 \text{ red counters}) - (3 \text{ blue counters})$$

First I tried subtracting simpler numbers, using counters: $8 - (-3)$.

$$(8 \text{ red counters} + 3 \text{ blue counters}) - (3 \text{ blue counters} + 3 \text{ red counters})$$

I used the zero principle to add enough blue and red counter pairs so that I could subtract.

$$(8 \text{ red counters} + 3 \text{ blue counters}) - (3 \text{ blue counters})$$

Then I subtracted -3 from both brackets. The answer is 11.

$$8 - (-3) = 8 + 3 \\ = 11$$

Subtracting the three negative counters has the same result as adding three positive counters.

$$58 - (-89) = 58 + 89 \\ = 147$$

Subtracting -89 would have the same result as adding 89, since I'd add 89 blue and 89 red counters, but subtract the 89 blue counters. $58 + 89$ is about $60 + 90$, or 150. The exact answer 147 makes sense.

The range is 147°C .

Reflecting

1. How does using counters and the zero principle show that subtracting $58 - (-89)$ has the same result as adding 58 and the opposite of -89 ?
2. Why does the order in which the numbers are written matter for subtraction but not for addition? Give examples to show your reasoning.

Work with the Math

Example 3: Subtracting using a calculator

Calculate $115 + (-218) - (-137)$.

Solution A: Using the negative sign key on a TI-15 calculator

$$115 + (-) 218 - (-) 137 \text{ Enter}$$



Solution B: Using the sign change key on other calculators

$$115 + 218 +/- - 137 +/- =$$



Example 4: Comparing differences

Par is the standard score for the number of strokes a good player usually needs to complete a hole or an entire game of golf.

Golfer	Tuesday score	Wednesday score
Rae	6 above par = +6	3 below par = -3
Lorna	2 below par = -2	1 below par = -1
Megan	5 above par = +5	par = 0

- a) By how many strokes did each golfer's score increase or decrease?
b) List the golfers in order, starting with the golfer whose score decreased the most.

Solution

$$\begin{aligned} \text{a) } -3 - 6 &= -3 + (-6) \\ &= -9 \end{aligned}$$

Rae's score decreased by 9 strokes.

$$\begin{aligned} -1 - (-2) &= -1 + 2 \\ &= 1 \end{aligned}$$

Lorna's score increased by 1 stroke.

$$\begin{aligned} 0 - 5 &= 0 + (-5) \\ &= -5 \end{aligned}$$

Megan's score decreased by 5 strokes.

- b) The order, starting with the golfer whose score decreased the most, is Rae, Megan, and Lorna.

A Checking

3. a) Draw a number line to represent $5 - (-2)$.
b) Use counters and the zero principle to represent $5 - (-2)$. Draw your model.
c) Write the addition statement that is equivalent to $5 - (-2)$.
4. The temperature was -8°C on Tuesday night and 2°C on Wednesday afternoon. What was the temperature change?

B Practising

5. Calculate. Order the differences from greatest to least.
- a) $7 - (-9)$ c) $-22 - (-10)$
b) $8 - 8$ d) $0 - 11$

6. Match the equivalent addition and subtraction expressions.

- a) $-6 - 9$ A. $10 + (-12)$
b) $-10 - (-12)$ B. $-6 + (-9)$
c) $6 - 9$ C. $-12 + (-10)$
d) $10 - 12$ D. $6 + (-9)$
e) $-12 - 10$ E. $-10 + 12$

7. Write each subtraction as an addition. Then calculate without using a calculator.

- a) $-10 - (-8)$ d) $0 - 10$
b) $3 - (-7)$ e) $-8 - 6$
c) $-6 - 0$ f) $-15 - (-5)$

8. Calculate. How could you estimate to see if each answer is reasonable?

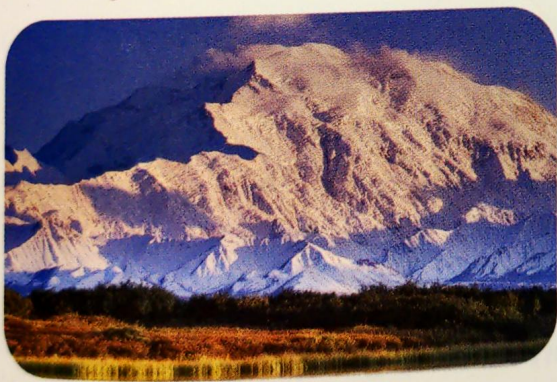
- a) $-32 - (-54)$ c) $19 - (-24)$
b) $36 - (-17)$ d) $-17 - 23$

9. For Example 3, Eva decided to estimate the answer for $115 + (-218) - (-137)$. She said, "About 100 minus about 200 is negative 100. Then add 137. My estimate is 37." Is her estimate reasonable? Explain.

10. Trevor recorded temperature changes for his science project. Copy and complete his chart.

Time	Temperature change ($^{\circ}\text{C}$)	Temperature ($^{\circ}\text{C}$)
Mon. a.m.	none	-7
Mon. p.m.	drop of 18	-25
Tues. a.m.	rise of 12	-13
Tues. p.m.	drop 10	-23
Wed. p.m.	rise 25	2

11. The highest point in North America is Mt. McKinley, and the lowest point is Death Valley. Mount McKinley is 6194 m above sea level. Death Valley is 87 m below sea level. Is 6100 m or 6300 m a better estimate of the difference between these heights? Explain.



12. Create a subtraction problem for each pair of integers.

a) -12 and 9

b) -1 and -14

13. Calculate.

a) $12 - (-5) + (-4)$

b) $-16 + 14 - 5$

c) $22 - 30 + 0 - (-7)$

d) $-40 + (-50) + 90 - (-60)$

14. Evaluate using a calculator.

a) $-128 - (-306)$

b) $-119 + (-237) - (-155)$

c) $-7145 - 3658 + (-2159)$

15. Erica had \$45. She spent \$18, earned \$22, and then spent \$6.

a) Write a number sentence, using only whole numbers, to show how much money Erica has.

b) Write another number sentence, using some negative integers, to show how much money Erica has.

16. a) What do you notice about subtracting 0 from an integer? Why does this make sense?

b) What do you notice about subtracting an integer from 0? Why does this make sense?

17. Dry ice is solid carbon dioxide. Its temperature is about -98°C . How much less is this temperature than room temperature, which is about 20°C ?

18. Suppose that you randomly select two of the following integers and then subtract the integers in the order you selected them. Explain whether the difference is likely to be positive or negative.

$-40, 3, -12, 83$

19. Determine the tenth term in each number pattern.

a) 12, 3, -6, ...

b) -18, -7, 4, ...

C Extending

20. When is the sum of two integers less than the difference?

21. When is the difference between two integers less than the sum?