

6.3

Exploring Integer Multiplication

You will need

- red and blue counters
- number lines
- coloured pencils
- a calculator

▶ **GOAL**

Explore models of, and patterns for, integer multiplication.

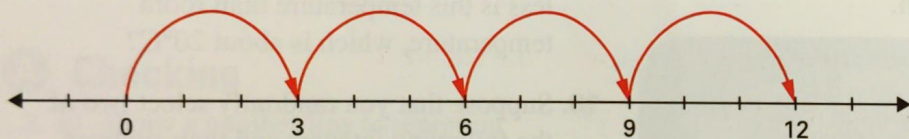
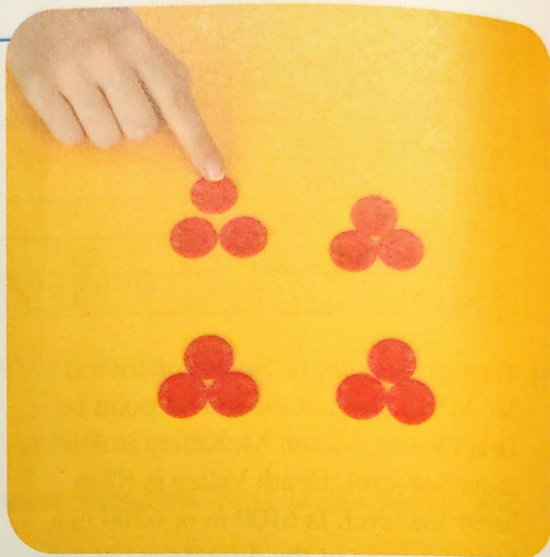
Explore the Math

Susan's younger brother Sam is sending a package of three stickers to each of his four cousins. He wonders how many stickers are in the four packages. Susan shows Sam how to multiply 4×3 .

"Multiplying 4×3 is the same as adding four groups of three."

Susan then shows Sam another way to multiply 4×3 .

"Start at 0 on a number line, and skip count by 3 four times."



Susan knows that the product of two positive integers is positive. She wonders what will happen if she multiplies by a negative integer; for example, $4 \times (-3)$.

? What happens when you multiply a positive or negative integer by a negative integer?

- Use blue counters to represent $4 \times (-3)$ as four groups of -3 . What is the value of the product?
- Start at 0 on a number line. Skip count by -3 s, going to the left, four times. Where do you end up?
- Use a calculator to verify your results in steps A and B.

D. Copy and complete the following chart.

Pattern 1	Pattern 2	Pattern 3	Pattern 4
$-2 \times 3 = \blacksquare$	$-5 \times 3 = \blacksquare$	$3 \times (-3) = \blacksquare$	$4 \times (-4) = \blacksquare$
$-2 \times 2 = \blacksquare$	$-5 \times 2 = \blacksquare$	$2 \times (-3) = \blacksquare$	$3 \times (-4) = \blacksquare$
$-2 \times 1 = \blacksquare$	$-5 \times 1 = \blacksquare$	$1 \times (-3) = \blacksquare$	$2 \times (-4) = \blacksquare$
$-2 \times 0 = \blacksquare$	$-5 \times 0 = \blacksquare$	$0 \times (-3) = \blacksquare$	$1 \times (-4) = \blacksquare$
$-2 \times (-1) = \blacksquare$	$-5 \times (-1) = \blacksquare$	$-1 \times (-3) = \blacksquare$	$0 \times (-4) = \blacksquare$
$-2 \times (-2) = \blacksquare$	$-5 \times (-2) = \blacksquare$	$-2 \times (-3) = \blacksquare$	$-1 \times (-4) = \blacksquare$
$-2 \times (-3) = \blacksquare$	$-5 \times (-3) = \blacksquare$	$-3 \times (-3) = \blacksquare$	$-2 \times (-4) = \blacksquare$

E. Use a calculator to verify the last product for each pattern in step D.

F. Each multiplication in Pattern 1 follows the pattern rule $-2 \times n$ or $-2n$, where n is any integer. Copy and complete this table of values.

Input integer (n)	Output integer ($-2n$)
3	$-2 \times 3 = -6$
2	$-2 \times 2 = \blacksquare$
1	$-2 \times 1 = \blacksquare$
0	$-2 \times 0 = \blacksquare$
-1	$-2 \times (-1) = \blacksquare$
-2	$-2 \times (-2) = \blacksquare$
-3	$-2 \times (-3) = \blacksquare$

G. Create a pattern rule for each of Pattern 2, Pattern 3, and Pattern 4.

H. Use your answers in steps D to G to complete this chart.

a	b	$a \times b$
positive	positive	
positive	negative	
negative	positive	
negative	negative	

Reflecting

- Use a real-world meaning for -3 to explain why $4 \times (-3) = -12$ makes sense.
- Why does it make sense that $4 \times (-3)$ and -3×4 have the same product?
- Why is it easier to model $4 \times (-3)$ than $-4 \times (-3)$ using counters?
- Explain your strategies for completing the chart in step H.

6.4

Multiplying Integers

▶ GOAL

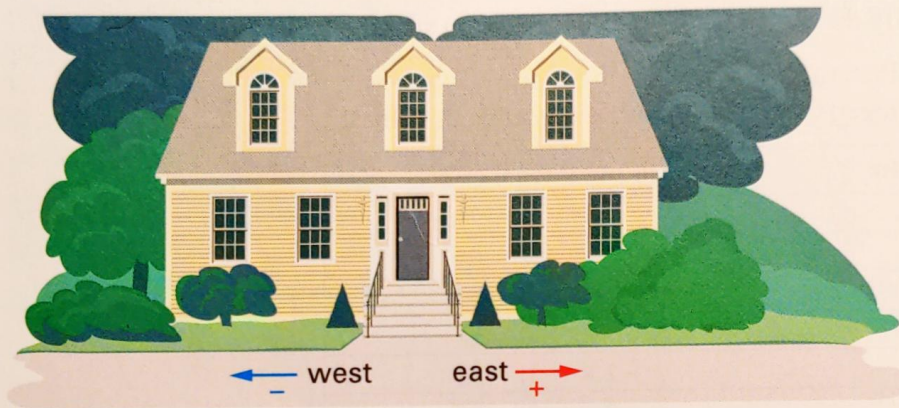
Develop and apply strategies to multiply integers.

You will need

- number lines
- coloured pencils
- red and blue counters
- a calculator

Learn about the Math

Eva says, “I can hike about 3 km/h. Sometimes I walk east from my home. Other times I walk west.”



? If you know where Eva is now, how can you calculate where she was earlier?

Use a positive sign to mean hiking to the east. Use a negative sign to mean hiking to the west. Home is neither positive nor negative because it is located at 0.

- How far from home is Eva if she hikes east for 10 h at 3 km/h? Draw this on a number line.
- Describe your drawing in step A using a repeated addition sentence. Then describe your drawing using a multiplication sentence.
- How far, and in which direction, has Eva hiked if her location can be determined using each of these calculations?
 - $4 \text{ h} \times 3 \text{ km/h}$
 - $2 \text{ h} \times (-3 \text{ km/h})$
- Model each multiplication in step C on a number line.
- Suppose that Eva has been hiking east for 6 h.
 - Why does $6 \text{ h} \times 3 \text{ km/h}$ represent her distance from home?
 - Why does $0 - 6 \text{ h} \times 3 \text{ km/h}$ represent how far, and in which direction, she would need to hike to get back home?
 - Why does $0 - 6 \text{ h} \times 3 \text{ km/h}$ have the same result as $-6 \text{ h} \times 3 \text{ km/h}$?

- F. Model each multiplication in step E on a number line.
- G. Suppose that Eva has been hiking west for 2 h.
- Why does $2 \text{ h} \times (-3 \text{ km/h})$ represent her distance from home?
 - Why does $0 - 2 \text{ h} \times (-3 \text{ km/h})$ represent how far she would need to hike to get back home?

Reflecting

- Why does it make sense to describe east as positive (+) and west as negative (-)?
- How does knowing $0 - 18 = -18$ show that $(-6) \times 3 = -18$ in step E?
- How does knowing $0 - (-6) = 6$ show that $(-2) \times (-3) = 6$ in step G?
- How can you predict the sign of the product of two integers?

Communication Tip

The multiplication of integers can be written without the multiplication symbol (\times). For example, $-10 \times (-20)$ can be written as $(-10)(-20)$.

Work with the Math

Example 1: Using integer products to solve distance problems

Eli walks to the west at 80 m/min.

- Where will he be after 5 min?
- Suppose that Eli walked west for 18 min. His position would be $18 \text{ min} \times (-80 \text{ m/min})$. How far must he walk to return to his starting position? What direction must he walk?

Denis's Solution

a) $5 \text{ min} \times (-80 \text{ m/min}) = -400$

Eli will be 400 m west.

I multiplied $5 \times 80 = 400$.

I know that a positive integer multiplied by a negative integer results in a negative integer. So the product must be -400 .

b) $18 \text{ min} \times (-80 \text{ m/min}) = 18 \times (-80)$
 $= -1440$

Eli must walk 1440 m east.

I multiplied $18 \times 80 = 1440$.

I know that a positive integer multiplied by a negative integer results in a negative integer. So the product must be -1440 .

Eli walked 1440 m west. He must walk back the same distance, but in the opposite direction, to return to his starting position.



Example 2: Modelling integer multiplication

Multiply $-3 \times (-2)$ using a model.

Eva's Solution: Using counters

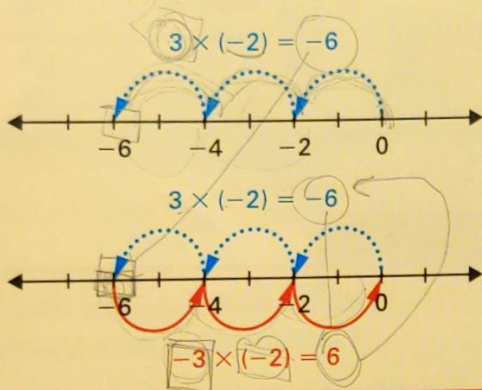
I can model $3 \times (-2)$ by adding three groups of two negative counters to 0. So, I can model $-3 \times (-2)$ by subtracting three groups of two negative counters from 0.

$0 - (\text{●● ●● ●● ●●})$ I can't take away the six negative counters. So I used the zero principle to add six positive counters and six negative counters.

$(\text{■● ■● ■● ■● ■● ■●}) - (\text{●● ●● ●● ●●})$ Then I subtracted.

$(\text{■● ■● ■● ■● ■● ■●}) - (\text{●● ●● ●● ●●})$ Six positive counters are left.
 $= (\text{■ ■ ■ ■ ■ ■})$ The answer is $-3 \times (-2) = 6$.

Nathan's Solution: Using a number line



$-3 \times (-2)$ means the same as $0 - 3 \times (-2)$, which is the opposite of $3 \times (-2)$.

To show $3 \times (-2)$, I drew 3 dotted blue arrows going left from 0, with each arrow 2 units. The arrows stop at -6 .

To show the opposite of $3 \times (-2)$, I drew 3 solid red arrows going right from -6 back to 0.

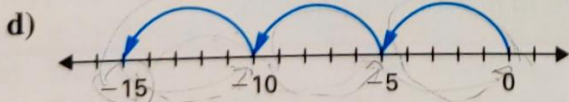
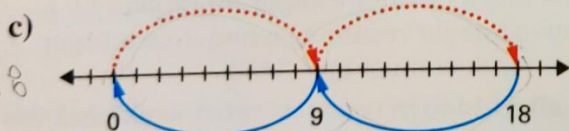
These red arrows go to the right 6 units. So the answer must be 6, which is the opposite of -6 .

A Checking

5. Write the multiplication sentence that each model represents.

a)

b) $0 - (\text{●● ●● ●● ●● ●● ●● ●● ●● ●● ●● ●● ●● ●● ●●})$



$3 \times (-5) = -15$

$2 \times 9 = 18$
 $-2 \times 9 = -18$

$-3 \times (-5) = 15$

6. Dario is on a cycling trip. He started at 0 km. He is now at position $20 \text{ h} \times (-20 \text{ km/h})$. When did he reach each of the following positions? Explain your reasoning.

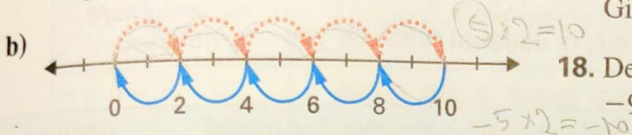
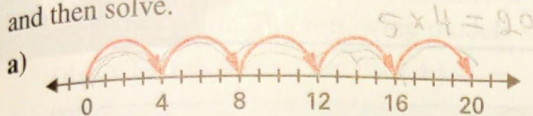
- $10 \text{ h} \times (-20 \text{ km/h})$
- $8 \text{ h} \times (-20 \text{ km/h})$
- 0 km

B Practising

- Model $-4 \times (-3)$ on a number line. Calculate the product. Explain what you did.
 - Model $-4 \times (-3)$ with counters. Calculate the product. Explain what you did.

8. How would you calculate $(-4)(-7)$? Justify your strategy.

9. Write each as a multiplication expression, and then solve.



10. Write an integer multiplication sentence for each description.

- a) Tyler rode a bus west for 4 h at 100 km/h.
- b) Jenna babysat for 3 h, earning \$5 an hour.
- c) The temperature fell 2°C a day for 6 days.

11. Jasmine has 50 shares of a stock. The value of each share went down by \$2 today. Express the total change in value of Jasmine's shares as an integer calculation.

12. Write a multiplication question for each repeated addition, and then solve.

- a) $-5 + (-5) + (-5)$
- b) $-8 + (-8) + (-8) + (-8) + (-8)$
- c) $0 + 0 + 0 + 0 + 0 + 0 + 0$

13. Write a repeated addition question for each multiplication, and then solve.

- a) $2 \times (-9)$
- b) $(4)(8)$
- c) $(3)(-6)$
- d) $(7)(0)$

14. Multiply.

- a) -2×4
- b) $-8 \times (-9)$
- c) $(7)(8)$
- d) $(-9)(-9)$

15. Calculate each product. Order the products from greatest to least.

- a) $(0)(-20)$
- b) $(-6)(-30)$
- c) $(7)(-80)$
- d) $(-20)(50)$

16. What is the greatest possible product of any two numbers in this list? Explain your answer.

- $-3, -7, -15, 6$

17. The product of two integers is between -20 and -25 . What are the integers? Give five answers.

18. Determine the missing integer for $-9 \times (\square) = 63$. Explain what you did.

19. Continue each pattern for the next three terms. Explain the pattern rule.

- a) $-2, 4, -8, 16, \square, \square, \square, \dots$
- b) $-15, 30, 90, -360, \square, \square, \square, \dots$

20. Multiply.

- a) $-5 \times 3 \times (-2)$
- b) $-2 \times (-3) \times (-4) \times (-5)$
- c) $4 \times (-3) \times (-2)$

21. The product of five different integers is -80 .

- a) What is the least possible sum of these five integers?
- b) Is it possible for the product of four different integers to be -80 ? Explain.

22. Estimate each product.

- a) $-35 \times (-25)$
- b) $(-18)(38)$
- c) $-21 \times 9 \times (-16)$

C Extending

23. Multiply any three integers that are not in the same row or column. Repeat with other sets of three integers. What do you notice?

-32	40	-24
28	-35	21
-8	10	-6

Mid-Chapter Review



Frequently Asked Questions

Q: How can you add integers?

A: You can use counters, a number line, or addition rules. Or you can use a calculator.

Addition question	Counter model	Number line model	Addition rule
$5 + 2 = 7$	$(\text{●●●●●}) + (\text{●●})$ $= \text{●●●●●●●}$		$5 + 2 = 7$
$-5 + (-2) = -7$	$(\text{●●●●●}) + (\text{●●})$ $= \text{●●●●●●●}$		$(-5) + (-2)$ $= -5 - 2$ $= -7$
$5 + (-2) = 3$	$(\text{●●●●●}) + (\text{●●})$ $= (\text{●●●●●●●})$ $= (\text{●●●●})$		$5 + (-2)$ $= 5 - 2$ $= 3$
$-5 + 2 = -3$	$(\text{●●●●●}) + (\text{●●})$ $= (\text{●●●●●●●})$ $= (\text{●●●●})$		$-5 + 2 = -3$

Q: How can you subtract integers?


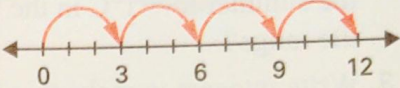

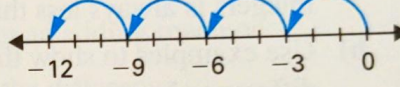
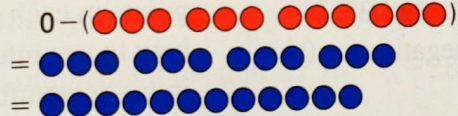
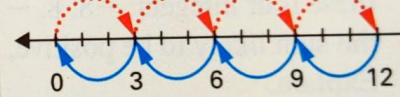
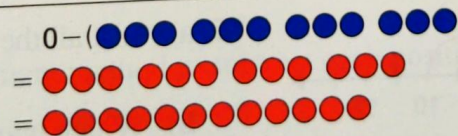
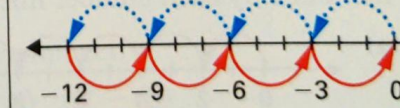
A: You can use counters or a number line, or add the opposite. Or you can use a calculator.

Subtraction question	Counter model	Number line model	Addition of opposite
$2 - 5 = -3$	$(\text{●●}) - (\text{●●●●●})$ $= (\text{●●●●●}) - (\text{●●●●●})$ $= \text{●●●}$		$2 - 5$ $= 2 + (-5)$ $= -3$
$-5 - (-2) = -3$	$(\text{●●●●●}) - (\text{●●})$ $= \text{●●●}$		$-5 - (-2)$ $= -5 + 2$ $= -3$
$5 - (-2) = 7$	$(\text{●●●●●}) - (\text{●●})$ $= (\text{●●●●●●●}) - (\text{●●})$ $= \text{●●●●●●●}$		$5 - (-2)$ $= 5 + 2$ $= 7$
$-5 - 2 = -7$	$(\text{●●●●●}) - (\text{●●})$ $= (\text{●●●●●●●}) - (\text{●●})$ $= \text{●●●●●●●}$		$-5 - 2$ $= -5 + (-2)$ $= -7$

Q: How can you multiply integers?

A: You can use counters, a number line, or repeated addition. Or you can use a calculator. The following models show that

$$(+)\times(+)=+ \quad (-)\times(+)= - \quad (+)\times(-)= - \quad (-)\times(-)= +$$

Multiplication question	Counter model	Number line model
$4 \times 3 = 12$	 <p>4 groups of 3 positive (red) counters give a total of 12 positive (red) counters.</p>	 <p>4 arrows to the right, with each arrow 3 units long, give a result of 12 units to the right, or 12. $3 + 3 + 3 + 3 = 12$</p>
$4 \times (-3) = -12$	 <p>4 groups of 3 negative (blue) counters give a total of 12 negative (blue) counters.</p>	 <p>4 arrows to the left, with each arrow 3 units long, give a result of 12 units to the left, or -12. $-3 + (-3) + (-3) + (-3) = -12$</p>
$-4 \times 3 = -12$	 <p>$0 - 4 \times 3$ means subtract 4 groups of 3 red counters from 0. Use the zero principle, and add 4 groups of 3 red counters and 4 groups of 3 blue counters. Then subtract the 4 groups of 3 red counters. You're left with 4 groups of 3 blue counters, or 12 blue counters.</p>	 <p>4 arrows to the left, with each arrow 3 units long, give a result of 12 units to the left, or -12. $0 - 4 \times 3$ means the opposite of 4×3, or the opposite of 12, which is -12.</p>
$-4 \times (-3) = 12$	 <p>$0 - 4 \times (-3)$ means subtract 4 groups of 3 negative (blue) counters from 0. The result is 4 groups of 3 positive (red) counters, or a total of 12 positive (red) counters.</p>	 <p>4 arrows to the right, with each arrow 3 units long, give a result of 12 units to the right, or 12. $0 - 4 \times (-3)$ means the opposite of $4 \times (-3)$, or the opposite of -12, which is 12.</p>

Practice Questions

1. Calculate. Order the results from least to greatest.

a) $-30 - 8$ c) $2 + (-2)$
 b) $4 - (-75)$ d) $39 - (-5)$

- (6.2) 2. Surface water temperatures on the Atlantic side of Newfoundland range from 13°C in the summer to -1°C in the winter. What is the range?

- (6.2) 3. Write integers to make this equation true. Give three possible answers.

$$\square - \square + \square = -10$$

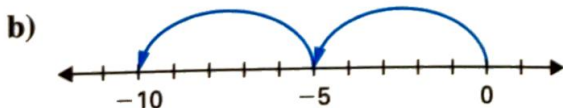
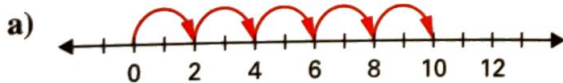
- (6.2) 4. a) Explain why the sum of two negative integers is always less than each integer.

- b) Use examples to show that the difference between two negative integers is not always less than either integer.

- (6.2) 5. Suppose that you randomly choose three of these four integers: -8 , 8 , -18 , and 28 . Is the sum likely to be positive, negative, or 0 ? Explain.

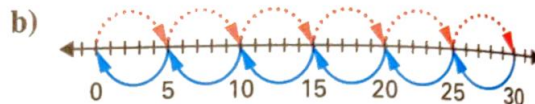
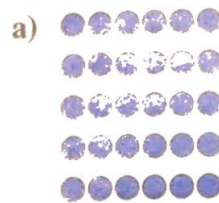
- (6.3) 6. Charlotte said, "If I multiply a negative integer and a positive integer, I know that the product is negative even without knowing the exact integers." Is Charlotte correct? Justify your answer.

- (6.3) 7. Write the multiplication equation that is modelled by each number line.



- (6.4) 8. Use a model to explain why $-5 \times 8 = -40$.

9. Write a multiplication equation for each model. (6.4)



10. Multiply. (6.4)

a) $(+5)(+9)$ c) $(-8)(+11)$
 b) $(-7)(-8)$ d) $0 \times (-12)$

11. Russell sold 40 shares of a stock. The value of each share has decreased by $\$1$ since he bought the stock. Express the amount Russell gained or lost as an integer. (6.4)

12. When playing a game, Matt lost eight points in each of his last three turns. Show how to use integers to determine the change in his score after these three turns. (6.4)

13. Is it possible for the product of two consecutive integers to be negative? Explain. (6.4)

14. The product of three consecutive integers is -1716 . What is the greatest of the three integers? (6.4)

15. a) List all the integer pairs of factors for -64 .
 b) Which pair has the greatest difference?
 c) Which pair has the greatest sum?
 d) Part (b) has two possible answers, but part (c) has only one answer. Explain why. (6.4)

16. Create and solve a problem for $8 \times (-10)$. (6.4)