6.5 Exploring Integer Division

You will need
- red and blue counters
- integer number lines
- coloured pencils

GOAL
Explore models and patterns for integer division.

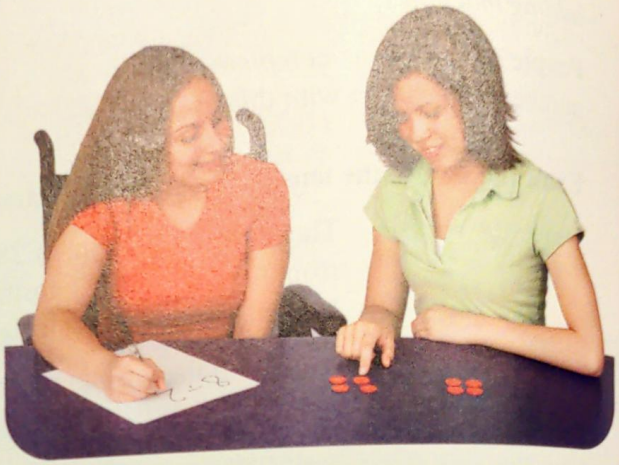
Explore the Math

Annika and Susan are talking about division.

Annika says, "If we have eight counters, you and I can share them equally."

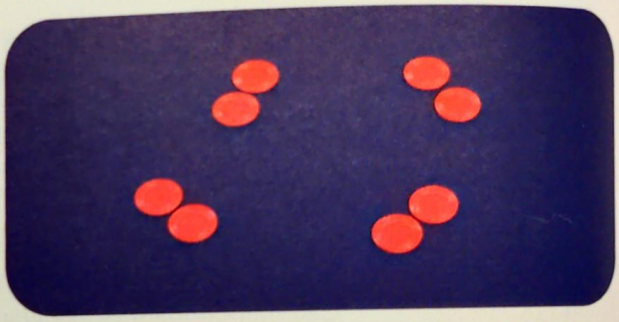
Susan says, "If two people share the eight counters equally, $8 \div 2$ is what each person gets. If we divide 8 into 2 groups, we have 4 in each group."

Then she says, "$8 \div 2$ also tells how many groups of 2 are in 8. If we divide 8 into groups of 2, we have 4 in each group."



Annika says, "Finding the answer to $8 \div 2$ is like finding what you need to multiply 2 by to get 8."

Susan wonders if she can use similar strategies to model what happens with negative integers.



? How can you use models for division to calculate $-8 \div (-2)$?

A. Division can mean finding the size of each part when you share an amount equally. For example, $-8 \div 2$ means sharing 8 negative counters equally between 2 parts. Use this meaning to model $-8 \div 2$ using counters.

B. Draw a blue arrow from 0 to -8 on a number line. Use the "sharing" meaning of division to model $-8 \div 2$.

C. Draw your models for steps A and B in a chart like the one below.

Division sentence	Counter model	Number line model
$-8 \div 2$		

D. Repeat steps A and B to show other ways that -8 can be shared equally among parts. Draw all the counter and number line models in your chart.

E. Division can also mean finding how many small groups of a certain size can be created from a total amount. For example, $-8 \div (-2)$ means dividing 8 negative counters into groups of 2 negative counters. Use this meaning to model $-8 \div (-2)$ using counters. Draw your model.

F. Draw a blue arrow from 0 to -8 on a number line. Use the "counting groups" meaning of division to model $-8 \div (-2)$.

G. Draw your models for steps E and F in a chart.

Division sentence	Counter model	Number line model
$-8 \div (-2)$		

H. Repeat steps E and F to show other ways that -8 can be divided into groups of equal value. Draw all the counter and number line models in your chart.

I. Copy the following chart. Fill in the products you already know. What do you notice? Use your reasoning to continue the patterns.

Pattern 1	Pattern 2	Pattern 3	Pattern 4
$-6 \div 2 =$ ■	$-9 \div 3 =$ ■	$-9 \div (-3) =$ ■	$-6 \div (-2) =$ ■
$-4 \div 2 =$ ■	$-6 \div 3 =$ ■	$-6 \div (-3) =$ ■	$-4 \div (-2) =$ ■
$-2 \div 2 =$ ■	$-3 \div 3 =$ ■	$-3 \div (-3) =$ ■	$-2 \div (-2) =$ ■
$0 \div 2 =$ ■	$0 \div 3 =$ ■	$0 \div (-3) =$ ■	$0 \div (-2) =$ ■
$2 \div 2 =$ ■	$3 \div 3 =$ ■	$3 \div (-3) =$ ■	$2 \div (-2) =$ ■
$4 \div 2 =$ ■	$6 \div 3 =$ ■	$6 \div (-3) =$ ■	$4 \div (-2) =$ ■
$6 \div 2 =$ ■	$9 \div 3 =$ ■	$9 \div (-3) =$ ■	$6 \div (-2) =$ ■

Reflecting

1. Why can sharing negative blue counters be used as a model for $-8 \div 2$, but not for $-8 \div (-2)$?

2. Why can you not use negative blue counters, or divide an arrow on a number line, to model $8 \div (-2)$?

3. How does the relationship between multiplication and division help to confirm the patterns in step I?

6.6 Dividing Integers

You will need
- a calculator
- a ruler

GOAL
Develop and apply strategies to divide integers.

Learn about the Math

Denis and Nathan keep track of changes in their weekly basketball scores. They record a positive change if a score goes up from the previous week, and a negative change if a score goes down. Their records for five weeks are given below.

Denis's Basketball Record

Week	2	3	4	5	6
Change in score	+5	−2	−2	−3	+7

Nathan's Basketball Record

Week	2	3	4	5	6
Change in score	−15	−8	−12	+20	+5

? What are Denis's and Nathan's mean weekly changes in score?

A. Determine the sum of each student's changes in score for the five-week period.

B. Why does it make sense that the mean change in score has the same sign as the total change in score?

C. Does the sign of the mean have to be the same as the sign that appears most often in the data? Explain how you know.

D. Calculate each student's mean change in score.

E. Multiply to check your division. Explain how this verifies your calculations in step D.

Communication Tip

Division can also be written in fraction form. The horizontal line means "divided by." For example, $48 \div (-12)$ can be written as $\frac{48}{-12}$, and $-48 \div 12$ can be written as $\frac{-48}{12}$.

Reflecting

1. How does thinking of $\frac{-10}{5}$ as the mean of five changes in score help to explain why a negative integer divided by a positive integer is negative?

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2. How can you use the relationship between multiplication and division to verify your answer in question 1?

3. Why can you solve any integer division equation by solving a related multiplication equation?

Work with the Math

Example 1: Using multiplication rules to predict the sign of a quotient

Determine the answer to $-12 \div (-4)$.

Susan's Solution

$-12 \div (-4) = \square$
$\square \times (-4) = -12$ — I wrote the multiplication sentence that relates to the division equation.

$3 \times (-4) = -12$ — I know that $3 \times 4 = 12$.

The answer is 3. — I also know that a positive integer multiplied by a negative integer results in a negative integer. So, the missing number must be 3.

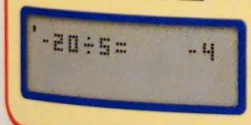
Example 2: Using a calculator to divide integers

Thomas checks the value of his stock at the end of each day. Calculate the mean change in the value of his stock over a five-day period.

Day	1	2	3	4	5
Total change in value ($)	+20	−40	−11	−1	+12

Denis's Solution:

20 [+] [(-)] 40 [+] [(-)] 11 [+] [(-)] 1 [+] 12 [Enter] [÷] 5 [Enter]



I calculated the mean by adding all the changes in value and then dividing the sum by 5.

A Checking

4. Divide. Multiply to check.

a) $-45 \div (-5)$

b) $\frac{0}{-8}$

c) $81 \div (-9)$

d) $\frac{-56}{7}$

5. Match each division equation with the related multiplication equation. Write the missing integers.

a) $-16 \div (-8) = \square$	A. $\square \times (-8) = 16$
b) $16 \div 8 = \square$	B. $\square \times (-8) = -16$
c) $-16 \div 8 = \square$	C. $\square \times 8 = 16$
d) $16 \div (-8) = \square$	D. $\square \times 8 = -16$

6. Andrea keeps track of changes in her weekly basketball scores. Her record for five weeks is given below.

Week	2	3	4	5	6
Change in score	+4	−11	−10	+15	−13

a) Use estimation to predict whether Andrea's mean weekly change will be positive or negative. Explain why.

b) Write a division equation to calculate Andrea's mean weekly change. What is Andrea's mean weekly change?

B Practising

7. Write the division equation represented by each model.

a) 

b) 

8. Write a multiplication equation for each division. Then solve the division.

a) $-72 \div (-9)$ c) $66 \div (-11)$

b) $84 \div 7$ d) $-800 \div 20$

9. Divide.

a) $40 \div (-5)$ d) $-121 \div (-11)$

b) $-24 \div 6$ e) $0 \div (-10)$

c) $\frac{-64}{-8}$ f) $\frac{54}{9}$

10. Why can $\frac{-8}{-2}$ not represent calculating a mean change in score?

11. Copy and complete using + or −. Record two answers for each expression.

a) ☐ 300 ÷ ☐ 15 is negative.

b) ☐ 300 ÷ ☐ 15 is positive.

12. Estimate each quotient.

a) $844 \div (-4)$ d) $-168 \div 8$

b) $-319 \div (-11)$ e) $136 \div (-17)$

c) $448 \div (-32)$ f) $-575 \div (-23)$

13. Determine each quotient. Multiply to check.

a) $\frac{48}{-12}$ d) $\frac{192}{-12}$

b) $\frac{-32}{8}$ e) $\frac{-256}{32}$

c) $\frac{-27}{-9}$ f) $\frac{-243}{-9}$

14. a) Copy and complete the charts.

a	b	$a \times b$	Example
+	+		
+	−		
−	+		
−	−		

a	b	$a \div b$	Example
+	+		
+	−		
−	+		
−	−		

b) How is determining the sign of a product the same as determining the sign of a quotient?

15. Determine the missing integer in each equation.

a) $40 \times$ ☐ $= -800$

b) ☐ $\times (-11) = -132$

c) $25 \times$ ☐ $= 2500$

d) ☐ $\times 24 = -192$

16. The quotient for $-35 \div 5$ is the opposite of the quotient for $-35 \div (-5)$. Why does this make sense?

17. Emma's scores for the first nine holes of a golf game are given below. Each positive integer represents a score above par. Each negative integer represents a score below par. What is Emma's mean score per hole?

Hole	1	2	3	4	5	6	7	8	9
Score	+1	−1	+3	+3	+2	0	0	−1	+2

18. Sanjay works in a recording studio. During a recent recording session, he noted the following decibel levels in a song:
−11 dB, −24 dB, +9 dB, +6 dB, +8 dB, −5 dB, +3 dB
a) What is the mean decibel level?
b) What is the difference between the mean decibel level and the lowest decibel level?
c) What is the difference between the mean decibel level and the highest decibel level?

19. What is the greatest integer quotient that can result from dividing one of these integers by another?
−120, −4, −15, −3

20. Write the next three terms.
a) 768, −384, 192, −96, ■, ■, ■, ...
b) −3645, 1215, −405, 135, ■, ■, ■, ...

21. How are multiplying integers and dividing integers similar? How are they different? Use examples to support your answers.

22. Calculate.
a) $-3 \times (-8) \div (-4)$
b) $\dfrac{(-6)(6)}{-4}$
c) $-63 \div (-7)(-9)$
d) $\dfrac{-144 \div 12}{-3}$
e) $(7)(-6) \div (3)(-7)$
f) $(-2)(-9) \div (2)(-3)$

23. Determine the missing integer.
a) $-49 \div ■ = -7$
b) $■ \div (-4) = 8$

24. Evaluate each expression when $x = -6$ and $y = 9$.
a) $-9x$
b) $-6y$
c) $5xy$
d) $6x + 7y$
e) $-8y - 5x$
f) $3xy \div y$

25. The Marianas Trench is the deepest spot in the oceans. It is located in the Pacific Ocean, just east of the Philippines. The maximum depth of the Marianas Trench is 10 962 m. The maximum depth of Lake Superior is 406 m. Create and solve an integer division question using this information.

C Extending

26. Suppose that you divided two integers. Then both integers are increased.
a) Can the quotient increase? Explain.
b) Can the quotient decrease? Explain.
c) Can the quotient be 0? Explain.

27. Suppose that you randomly choose an integer value between −80 and −90 for a and an integer value between −5 and −8 for b. What is the probability that $\frac{a}{b}$ is also an integer?

28. The product of three integers is negative. Suppose that you multiply this product by two other integers. If the final product is positive, what do you know about the other two integers?

29. Suppose that you divide integer a by integer b, and the result is negative. Then you divide integer a by an integer that is 2 greater than b. The result is positive. What did you divide by the first time?

6.7 Order of Operations with Integers

You will need
- a calculator

GOAL
Apply the rules for the order of operations with integers.

Learn about the Math

Suppose that you win a contest, but you have to answer the following skill-testing question before you can claim the prize:

$$\frac{6 \div (-3) + [(4 - (-5)) \times (-7)]}{4 - 5}$$

? What is the answer to the skill-testing question?

If there are brackets within brackets, or nested operations, perform the nested operations in the innermost brackets first.

When there is a dividing line separating the numerator of an expression from the denominator, calculate the value of the numerator, then the value of the denominator, and finally divide.

Communication Tip

- Different types of brackets can be used to make it easier to match beginning and end brackets.
- An expression that is written in fraction form can be evaluated by dividing the final value of the numerator by the final value of the denominator.

Example 1: Evaluating an expression in fraction form

Use the order of operations to evaluate the skill-testing question.

Annika's Solution

$$\frac{6 \div (-3) + [(4 - (-5)) \times (-7)]}{4 - 5}$$

I started by determining the value of the numerator.

I calculated what's in the innermost brackets.

$$= \frac{6 \div (-3) + [9 \times (-7)]}{4 - 5}$$

I calculated what's in the square brackets.

$$= \frac{6 \div (-3) + [-63]}{4 - 5}$$

I divided.

$$= \frac{-2 + [-63]}{4 - 5}$$

I added to calculate the numerator.

$$= \frac{-65}{4 - 5}$$

I subtracted to calculate the denominator.

$$= \frac{-65}{-1}$$

I divided the numerator by the denominator.

$$= 65$$

The answer is 65.

Example 2: Performing different calculations in the same step

How can you calculate the answer to the skill-testing question more efficiently?

Eva's Solution

$6 \div (-3) + [(4 - (-5)) \times (-7)]$

I calculated the numerator first. I did three steps at the same time, since they don't interfere with each other.

$6 \div (-3) = -2$, $4 - (-5) = 9$, and $9 \times (-7) = -63$

$-2 + (-63) = -65$

The numerator is -65.

$4 - 5 = -1$

Then I calculated the denominator. It's like an expression in brackets.

$-65 \div (-1) = 65$

I know that a fraction can represent division. I also know that a negative integer divided by a negative integer is positive.

Reflecting

1. What advantages does Eva's method of evaluating the expression have over Annika's method?
2. Nathan says, "If an expression has a numerator and a denominator, like $\frac{-6 + (-10)}{(-4)(2)}$, the last calculation is division." Is Nathan correct? Explain.
3. Why is it important to use the rules for the order of operations when you calculate the value of an integer expression?

Work with the Math

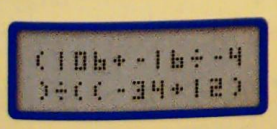
Example 3: Using a calculator to evaluate an expression

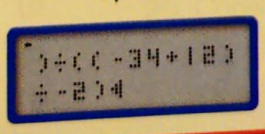
Calculate $\frac{106 + (-16) \div (-4)}{(-34 + 12) \div (-2)}$.

Solution

(106 + (-) 16 ÷ (-) 4) ÷
(((-) 34 + 12) ÷ (-) 2) Enter

Use the bracket keys, (and), to separate the numerator from the denominator and show the operation of division.





A Checking

4. Calculate.

a) $-9 - (-6) \div 6$

b) $4 \times (-8) - (-5)$

c) $-8 \times (-3) - (-8) \div (-4)$

d) $-16 \div [-2 - (-18)] \times (-1)$

B Practising

5. In each expression, which calculation(s) should you do first?

a) $-5 + (-6) \times (-8) \div 2$

b) $-8 \times 6 \div (-2) - [-9 \times (-3)]$

6. Calculate.

a) $-2 + (-3) \times (-8 + 4)$

b) $-9 - (-8) \times 7 + [6 \times (-2)]$

c) $7 \times [8 - (-2) \times (-6)]$

d) $-6 \div (-3) - [-8 \div (-2)]$

e) $0 + (-4) - 7 \times 5$

f) $[-14 + (-23)] - [(-17 - 2) \times 10]$

7. Calculate.

a) $7 \times [-3 - (-5)] \times 8$

b) $-3 - (-4) \times [2 \times (-6)]$

c) $-15 \div (-3) + 2 \times (-8)$

d) $[-2 - (-8)] \times (-5)$

e) $35 + (-4) \times (-8) - 7$

f) $18 \times (-3 - [8 \times (-5)])$

8. There is an error in this solution.

$$\begin{aligned} 3 \times (-8) \div (-2 - 4) &= -24 \div (-2 - 4) \\ &= 12 - 4 \\ &= 8 \end{aligned}$$

a) Find the error.

b) Explain how to correct the error.

9. Calculate.

a) $\dfrac{-6 + (-10)}{(-4)(2)}$

b) $\dfrac{49 \div (-7)}{1 + (-2)(-3)}$

c) $\dfrac{28 \div (-4 - 3)}{(-2 + 4) \times 2}$

d) $\dfrac{27 + (-18) \div (-2)}{(-2 + 5)(-2)}$

e) $\dfrac{-9 + (-16) - 10}{(-7)(10) \div (-2)}$

f) $\dfrac{[6 + (-38)] \div 4(-2)}{(-2 + 4)(5 - 6)}$

10. Create an integer expression that shows why the rules for the order of operations are needed. Explain how your expression shows this.

11. Using brackets, group the terms in this expression to get the least possible result.

$40 \times 6 - 3 \times 4 - 5$

12. a) Evaluate with a calculator.

$-147 + 156 \div (-4) + 405 \div (-15)$

b) Does your calculator follow the order of operations? Explain how you know.

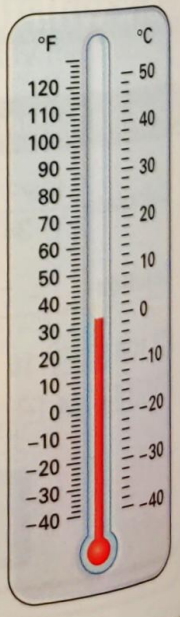
13. The formula for converting temperatures from Fahrenheit (F) to Celsius (C) is $C = (F - 32) \times 5 \div 9$. Use the formula to calculate each temperature in degrees Celsius.

a) 32°F

b) 212°F

c) −4°F

d) −40°F



14. This chart shows the predicted high temperatures in Iqaluit for a week in November. Use an integer expression to determine the mean predicted high temperature for the week.

Day	Predicted high temperature (°C)
Mon.	−4
Tues.	−4
Wed.	0
Thurs.	1
Fri.	−1
Sat.	−2
Sun.	−4

15. Two sisters bought some shares in four stocks with money they earned cutting lawns. This chart shows how their stocks changed in value over one month. Write an integer expression that could be used to determine the change in the total value of their stocks. Evaluate your expression.

Stock	A	B	C	D
Number of shares	10	100	50	30
Value of each share Aug. 1 ($)	42	5	38	19
Value of each share Sept. 1 ($)	39	4	42	21

16. Copy each equation. Fill in the missing operation signs.

a) $36 \square (4 \square 1) \square 2 = 24$

b) $-12 \square 4 \square (-3) = -24$

c) $-15 \square (-12) \square 6 \square 16 = -47$

17. The price of gold changes daily. One week, the price started at $350 per ounce on Monday and changed −$2 each day for 3 days, and then +$8 each day for the next 2 days.

a) Copy and complete the chart based on the data given.

Day	Starting price ($)	Final price ($)	Change in price ($)
Mon.	350		
Tues.			
Wed.			
Thurs.			
Fri.			

b) Calculate the mean final price of gold for the week.

c) Calculate the mean change in price for the week.

18. Evaluate each expression if $x = -2$, $y = 4$, and $z = -6$.

a) $\dfrac{2x + 7y}{z}$

b) $\dfrac{-4xz}{y^2}$

c) $\dfrac{y^2 - z^2}{x^2}$

C Extending

19. Calculate.

a) $(-3)^2 + (-8) \div (-2)$

b) $-3 \times [-4 + 2^3]$

c) $\dfrac{(-2 + 10)}{2^2}$

d) $\dfrac{-5 + (-3)(-6)}{(-2)^2 + (-3)^2}$

20. Copy and complete the equation using each digit from 1 to 5 once.

$\square \times \square\square - (-\square) + (-\square) = 50$